lunedì 10 dicembre 2018 11:58

POPOLAZIONE $X(\mu; \sigma^2)$

$\downarrow$

$n = 2 : X_1, X_2$

$T_1 = \frac{X_1 - X_2}{2}$ $\qquad T_2 = \frac{X_1 + X_2}{2} = \bar{X} \nearrow \mu$

STIMATORI di μ $\qquad n = 2$

$T \to \theta$

$M.S.E.(T) = DIST^2(T) + VAR(T)$

$E(T) = \theta$: T CORRETTO

$DIST(T) = D(T) = \theta - E(T)$

$E(T) \neq \theta$

$E(T_1) = \frac{1}{2}[E(X_1) - E(X_2)] =$
$= \frac{1}{2}[\mu - \mu] = 0 \neq \mu$

$D(T) = \mu - 0 = \mu \leftarrow T_1$ DISTORTO

$VAR(T_1) = V\left[\frac{1}{2}X_1 + \left(-\frac{1}{2}\right)X_2\right] =$
$= \frac{1}{4}V(X_1) + \frac{1}{4}V(X_2) =$
$= \frac{1}{4}\sigma^2 + \frac{1}{4}\sigma^2 = \frac{1}{2}\sigma^2$

$E(T_2) = \frac{1}{2}[E(X_1) + E(X_2)] =$
$= \frac{1}{2}[\mu + \mu] = \mu$

T_2 CORRETTO

$VAR(T_2) = VAR\left[\frac{1}{2}X_1 + \frac{1}{2}X_2\right] =$
$= \frac{1}{4}\sigma^2 + \frac{1}{4}\sigma^2 = \frac{1}{2}\sigma^2$

$M.S.E.(T_1) = D^2(T_1) + V(T_1) =$
$= \mu^2 + \frac{\sigma^2}{2}$

$>$

$M.S.E.(T_2) = D^2(T_2) + V(T_2) =$
$= 0 + \frac{\sigma^2}{2}$

T_2 è più efficiente

$n : \boxed{\frac{\sum_{i=1}^{n} X_i}{n} = \bar{X}} \to \mu$

$n = 2 : \frac{\sum_{i=1}^{2} X_i}{2} = \bar{X}$

$V(\bar{X}) = \frac{\sigma^2}{n}$

POPOLAZIONE X

$\mu \leftarrow \underset{V.C.}{\bar{X}} : E(\bar{X}) = \mu_X ; \sigma^2_{\bar{X}} = \frac{\sigma^2_X}{n}$

POPOLAZIONE $\underset{V.C.}{X} : E(X) = \mu ; \sigma^2_X = VAR(X)$

POPOLAZIONE

$\downarrow$

$n \quad X_1, X_2, \dots, X_n$ INDIP.

$\left.\begin{matrix} T_1 \\ T_2 \end{matrix}\right\}$ STIMATORI di μ : $E(T)=\mu$.

$E(T_1)=E\left(\frac{\sum X_i}{n}\right)=E(\overline{X})=\mu$ CORRETTO

$E(T_2)=\frac{1}{2}\left[E(X_1)+E(X_n)\right]=\frac{1}{2}(\mu+\mu)=\mu$ CORRETTO

$V(T_1)=V(\overline{X})=\frac{\sigma^2}{n}$

$V(T_2)=V\left[\frac{1}{2}X_1+\frac{1}{2}X_n\right]=$
$=\frac{1}{4}V(X_1)+\frac{1}{4}V(X_n)=\frac{1}{4}2\sigma^2=\frac{\sigma^2}{2}$

$m.s.e.(T_1)=D^2(T_1)+V(T_1)=$
$=0+\frac{\sigma^2}{n}=$
$=\frac{\sigma^2}{n}$

$<$ per $n>2$

$m.s.e.(T_2)=D^2(T_2)+V(T_2)=$
$=0+\frac{\sigma^2}{2}=$
$=\frac{\sigma^2}{2}$

T_1 è meglio di T_2

$X : \underset{}{\overset{v.c.}{R}}=2X+13 \longleftarrow \underset{v.c.}{T}=2\cdot\overline{X}+13$ $\boxed{E(T)=\mu_R}$?

$\boxed{T \text{ è CORRETTO per } \mu_R}$

$\downarrow \quad \mu_R$

$n=12 \quad X_1=\dots;\dots;X_{12}=\dots \Big\}$ $\overline{X}_n=\overline{X}$ MEDIA CAMPIONARIA

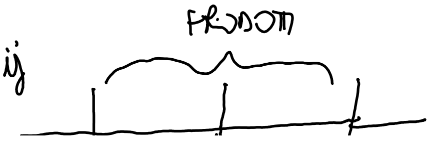
$\mu=E(X)$ $\qquad E(\overline{X})=\mu$

$\mu_R=E(R)=E(13+2X)=13+2\cdot E(X)=\boxed{13+2\cdot\mu}$

$E(T)=E(13+2\overline{X})=13+2\cdot E(\overline{X})=\boxed{13+2\cdot\mu}$

$\Rightarrow$ T CORRETTO

n_{ij}



566 CLIENTI ≡ campione

$\overset{\shortparallel}{N} = \sum\sum n_{ij}$

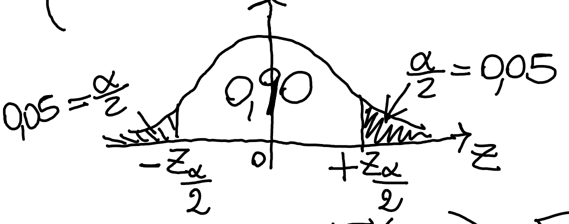
$\hat{p}$

$$P\{\ldots < p < \ldots\} = 0{,}90$$

$\hat{p}$ FREQUENZA RELATIVA CAMPIONARIA $\quad \hat{p} = \frac{265}{312} = \frac{x}{n} = 0{,}849$

$$(1-\hat{p}) = 1 - 0{,}849 = 0{,}151$$

$$P\left\{\hat{p} - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right\} = 1 - \alpha = 0{,}90$$
$$= 1 - 0{,}10$$



$z = 1{,}64$

$0{,}9495$

$$F\left(+z_{\frac{\alpha}{2}}\right) = P\left(Z < +z_{\frac{\alpha}{2}}\right) = \int_{-\infty}^{+z_{\frac{\alpha}{2}}} f(z)\,dz = 1 - 0{,}05 = 0{,}95$$

$0{,}9505$

$z = 1{,}65$

$$+z_{\frac{\alpha}{2}} = \frac{1{,}64 + 1{,}65}{2} = 1{,}645$$

$$-z_{\frac{\alpha}{2}} = -1{,}645$$

$$P\left\{0{,}849 - 1{,}645 \cdot \sqrt{\frac{(0{,}849)(0{,}151)}{312}} < p < 0{,}849 + 1{,}645 \cdot \sqrt{\frac{(0{,}849)(0{,}151)}{312}}\right\} = 0{,}90$$

$$P\{0{,}816 < p < 0{,}882\} = 0{,}90$$

$$P\{\ldots < p < \ldots\} = 0{,}95 = 1 - \alpha = 1 - 0{,}05 \quad \left(\frac{\alpha}{2} = 0{,}025\right)$$

$\hat{p} = 0{,}5 \qquad$ Δ ampiezza $= 0{,}6 \qquad n = ?$

$\hat{p} = 0{,}5$ Ampiezza $= 0{,}6$ $(n = ?)$

Ampiezza $= \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} - \left(\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) =$

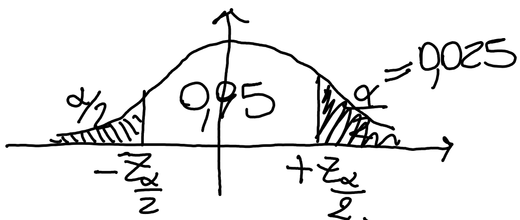
$$= \cancel{\hat{p}} + z_{\frac{\alpha}{2}}\sqrt{\dots} - \cancel{\hat{p}} + z_{\frac{\alpha}{2}}\sqrt{\dots} = \boxed{2\cdot z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0{,}6}$$

$$0{,}6 = 2\cdot z_{\frac{\alpha}{2}}\cdot\sqrt{\frac{(0{,}5)(0{,}5)}{n}}$$

$$\boxed{0{,}6 = 2\cdot(1{,}96)\cdot\sqrt{\frac{(0{,}5)(0{,}5)}{n}}}$$

$\downarrow$

$n = 10{,}67\ldots \cong 11$



$F(+z_{\frac{\alpha}{2}}) = 1 - 0{,}025 = 0{,}975$

$\Vert$

$+z_{\frac{\alpha}{2}} = +1{,}96$

$X: \longrightarrow n\ X_1, \dots, X_n$

θ: $= \mu$, $= \sigma^2$, $= p$

T: $\bar{X} = \frac{\sum X_i}{n}$, $S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})}{n-1}$, $\hat{p} = \frac{x}{n}$

CORRETTO? $E(T) = \theta$

$E(\bar{X}) = \mu$
$E(S^2) = \sigma^2$
$E(\hat{p}) = p$

T_4

stim. $T_3 = \frac{X_1 - 3X_2 + 2X_3}{3}$ $\quad E\left(\frac{X_1 - 3X_2 + 5X_3}{3}\right) = \mu$

$\uparrow$

param. σ^2 $\quad E(T_3) = \sigma^2$?

μ

$E(T_3) = \mu$

$T \quad \theta$

$+3X_4$

$E(T_3) = \frac{1}{3}\left[E(X_1) - 3E(X_2) + 2E(X_3)\right] =$

$\frac{1}{3}[\mu - 3\mu + 2\mu] = 0 \neq \mu$

$\bar{T}$ θ

$$= \frac{1}{3}\left[\mu - 3\mu + 2\mu\right] = 0 \neq \mu$$

$0 + 3\mu$ $\quad T_3$ DISTORTO

Intervalli di confidenza per μ

$$\bar{X} \begin{cases} \sim v.c.\, N & \left[P\left\{ \bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + \dots \right\} = 1-\alpha \right. \\ \sim v.c.\, t & P\left\{ \bar{x} - t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \right\} = 1-\alpha \end{cases}$$

$P\{$

$X \sim v.c.\, N \Rightarrow \bar{X} \sim v.c.\, N$

$\bar{X}$ v.c. $\; n \to +\infty \; (n > 30)$ v.c. $\bar{X} \sim v.c.\, N$

$$\boxed{\begin{matrix} n < 30 \\ X \sim v.c.\, N \\ \sigma^2 = ? \end{matrix}} \Rightarrow \bar{X} \sim v.c.\, t$$

$1 - \alpha = 0{,}90$

$\alpha = 0{,}10$

$$\boxed{\begin{matrix} n > 30 \\ n \to +\infty \\ X \sim v.c.\, N \\ \sigma^2 = ? \end{matrix}}$$

$n - 1 = \nu$

$$P\left\{ \bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < \bar{x} + \dots \right\}$$

$z =$

$t =$

$n-1=\nu$	area
.	
.	t
.	

$$\lim_{n \to +\infty} P(|\theta - T| < \varepsilon) = 1$$

$\alpha = \beta$

area $= P(t > t_0)$